Controlling mathematics by verbalizing mathematical concepts

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Abstract

The objective of the present study was to identify one or more common critical incidents that are decisive for success or failure at mathematical exams. First year business school students who had failed their mathematical exams were interviewed. Data were analyzed by Critical Incident Technique to uncover possible common causes of failure. Our results showed that the students who had failed the mathematical exam could hardly verbalize anything with regard to mathematical concepts. It became clear that they did not understand the concepts. This lack of understanding gave rise to lack of self-control and thereby anxiety. The students had practiced mathematical operations, but had never reflected upon or talked about mathematical concepts. Without conceptual understanding, the content of exam questions was not recognized. By this the students were neither able to decide which mathematical rules to be used nor the correct order of operations.

Key words: Mathematical concepts, Verbalization, Control, Self-efficacy, Anxiety

Introduction

It is well known that a substantial number of students struggle with learning mathematics. The consequence is a general high percentage of failure at mathematics exams. The objective of the present study was to identify one or a few common critical incidents that are decisive for success or failure at mathematical exams. The critical Incident Technique (CIT) (Chell, 2004) was used to identify the incidents that determine passing or failing at mathematical exams. By incident is meant any specifiable human activity that is sufficiently complete to permit inferences and predictions about the person performing the activity. To be critical, the incident must occur such that there is a clear connection to the consequence of the incident (Chell 2004).

Mathematics anxiety is shown to inhibit math achievements (Ashcraft and Kirk 2001, Beilock and Carr 2011, Lyons and Beilock 2012). Underperformance in mathematics has been explained by the phenomenon of "stereotype threat" (Schmader and Johns 2003) and "choking under pressure" (Baumeister 1984, Beilock et al. 2004). Stereotype threat is defined as a burden that makes people confirm cultural stereotypes reducing their intellectual and academic abilities (Steele and Aronson 1995). "Choking" refers to performance decrements under pressuring circumstances (Baumeister 1984). By pressure is meant any factor that increases the importance of performing well on a particular occasion. "Choking under pressure" may be defined as a kind of "performance anxiety". In spite of focusing on somewhat different phenomena, the conclusions from the above referred studies on mathematics were similar and involved working memory. The working memory is a short term memory that maintains a certain amount of information with immediate relevance to the task at hand while preventing distractions from the surroundings as well as irrelevant thoughts. If the working memory is disrupted, performance may suffer. As discussed by Beilock and Carr (2011), anxiety generates intrusive worries about the situation that disturb the working memory and reduce the capacity normally devoted to skill execution. Further by Beilock and Carr (2011), research indicates that the unpleasant nature of anxiety reduces the capacity of the working memory to recognize and understand any verbal information as e.g. task information. Also the stereotype threat reduces the performance of the working memory. Finally, choking pressure creates mental distractions that take over a part of the working memory capacity that would normally be devoted to skill execution.

Lyons and Beilock (2012) identified neural areas involved in math anxiety. Further, they found that the negative relation typically seen between math anxiety and math competence arises before math performance starts. Based on these results, they proposed to help students to learn how to take cognitive control over the math-related anxiety response before it started to dominate.

The studies referred to above, constituted a background for the present explorative study. The referred studies confirm that anxiety causes underperformance in mathematics. Our purpose, however, was to explore the concrete mechanism(s) behind the mathematics anxiety in the perspective of educational implications.

Method

Data were gathered by largely unstructured interviews and analyzed by Critical Incident Technique (CIT) (Chell 2004) and Grounded theory approach (Strauss and Corbin 1998). The intention was to capture the thoughts, feelings, and external circumstances that influenced the failure in mathematics. CIT is a qualitative method for investigation of significant incidents with the purpose of solving practical problems and developing psychological theories. Sixteen business school students who had failed their first year's mathematical exams were interviewed. They were invited to the interview by mail. Each interview lasted for about one and a half hour. The interview guide consisted of three essential questions: (i) Could you please tell me a little bit about why you think you failed your mathematic exam, (ii) How did you experience to fail, and (iii) What do you need in order to succeed in mathematics? These questions were followed up by extensive probing. In addition, two student advisors in mathematics were interviewed to explore their perspective on why some students fail in mathematics. The two advisors, who both had top marks in mathematics, were asked to explain the mechanisms behind lack of success in mathematics.

The interviews had the hall-mark of being a dialogue more than a one-way question and answer interview. A dialogue about specific mathematical concepts was introduced at the end of the interviews. The purpose was to explore concretely how mathematical concepts were reacted upon and verbalized.

The analytic process was based on grounded theory approach (Chell 2004, Strauss and Corbin 1998). The grounded theory approach involves open, axial and selective coding (Strauss and

Corbin 1998). By open coding, tentative codes or concepts for chunks of relevant data were created. The axial coding involves identifying relationships among the open codes, which were clustered into more integrative concepts or categories. The core variable including all the relevant codes was constructed by selective coding. Constant comparative approach was applied, i.e. between codes, concepts, categories, and individuals. When applying a grounded theory approach, reduction of preconceptions is necessary. The combination of grounded theory with CIT, however, makes it important to keep in mind the objective of the study which is to identify one or more critical incidents.

Results

Some students started out by saying that they were not good at mathematics even before the first question. "*I'm not strong at math.*" "*I've always struggled with math.*" "*I can't manage math*." Such general negative self-talk (Tice 1997), was most striking at the start of the interviews. The respondents did only moderately blame previous teachers or current lecturers for their failure. The students spoke about having practiced too little, about practicing but not managing, about bad teachers, about not understanding, about anxiety for appearing stupid, about feeling humiliated by the lecturer and fellow students, about frightening formulas, about procedures which seemed nearly identical, about not knowing when the calculation was finished, about not having any students to collaborate with, about lecturing going too fast, about losing the continuity, about not daring to ask, about leafing through the book without recognizing, about not understanding, performance anxiety, procrastination, and personal problems. These expressions are examples of open codes. The students also expressed that they did not manage to solve mathematical tasks which deviated the slightest from the ones that had been demonstrated by the lecturer or shown in the text book. The respective open code was designated "lack of transference". Concerning the third question about their needs in order to succeed in mathematics, they talked about the lecturer progressing too fast, need for repetition, someone to talk with without being afraid of appearing stupid, about not knowing anybody, and about no one inviting them into groups. Here the open codes were "need for repetition", "need for time", "need for talk", and "need for at least one mathematical friend".

Examples of categories from the axial coding are "control", "mathematical discussions", self-efficacy, "performance anxiety", and "need for talking". Largely, the students expressed identical problems concerning mathematics. The expressed problems, however, could not be described as critical incidents, but more as consequences of critical incidents not mentioned. I therefore asked the respondents to describe the sequence of events that had happened with respect to mathematics from primary school to the current situation. Such a sequential description is in harmony with the recommendation of Chell (2004) when respondents appear not to be able to identify critical incidents themselves. They often talked about not having basic knowledge as a causal problem. This lack of basic knowledge was exemplified as not knowing when the exercises started or stopped, not recognizing what was asked for, what kind of rules to be used, and uncertainty in general.

After having performed and analysed the first two interviews, it became apparent that the respective students were neither able to express the meaning of mathematical concepts nor the connections between them. It also became clear that they had never discussed or verbalized mathematics professionally with anyone. After this preliminary result, I included some concrete questions at the end of the interview aimed at exploration of the students' understanding of concrete mathematical concepts such as subtraction, addition, multiplication, deviation, equation, function and derivation. Both the understanding/definition of the individual concept as well as the relationship between them was highlighted. The preliminary result from the first two interviews that the respondents were neither able to express the meaning of the concepts nor the connections between them, were to the fullest confirmed. It should be emphasized that the students were generally eloquent.

The interviews revealed that students, who had failed the mathematic exams, had never spoken the language of mathematics. How do we speak the language of mathematics, they replied. I told them that we are speaking mathematics when we delve with a concept, e.g. the concept of "equation". I exemplified elements of a dialog about the concept of equation: "What do you understand with the concept of equation? What is the difference between an equation and an inequality, or between an equation and a function? Could an equation be termed equality?" The students having failed in mathematics were not able to reflect loudly on these questions. They expressed that they had never participated in such a dialog.

It is quite widespread to use mind maps as a study tool. None of the students had used this study tool in mathematics. Several students expressed, however, that they used mind maps as a tool in other courses. As a part of the interview, I asked them to start on a mind map placing the concepts of addition and subtraction relative to each other. All of the respondents placed these concepts as oppositional. I asked if addition and subtraction could be seen in another way, e.g. as the same operation. It seemed that the majority of students got a eureka moment when they understood that subtraction is the same as the addition of a negative number. The effect of communication about mathematical concepts and problems, *per se* seemed to give them enhanced understanding of the concept.

The open coding of the interviews of the two student advisors gave codes as obsessive thoughts concerning maths, attendance, math as a love-hate subject, slow starters, understanding, decision to go for it, low self-efficacy, and anxiety, especially for long mathematical expressions. The axial codes concerned understanding, will-power and self-efficacy. These codes concerned the students who were advised and not the ones advising.

The selective coding of the grounded theory approach indicated a limited understanding of mathematical concepts. This limited understanding was manifested as not being able to define or explain mathematical concept orally. Further, this consequence gave rise to a feeling of lack of control and thereby of anxiety, or even panic at the exam.

Discussion

Conceptual knowledge has been defined as explicit or implicit understanding of (i) the principles that govern a domain and (ii) the interrelations between pieces of knowledge in a domain (Rittle-Johnson and Alibali 1999). Rittle-Johnson and Alibali (1999) defined procedural knowledge as a sequence of actions for solving problems. According to the authors, these two types of knowledge lie on a continuum and cannot always be separated. The two ends of the continuum, however, represent two different types of knowledge. Research on mathematics learning supports the idea that conceptual understanding plays a significant role in learning of procedures (Rittle-Johnson and Alibali 1999). Further, conceptual knowledge may have greater influence on procedural knowledge that the reverse (Rittle-Johnson and Alibali 1999).

We identified “scant understanding of mathematical concepts” as the critical incident behind failing at mathematical exams. A manifestation of this low understanding was that the students were not able to solve mathematical problems that had a slightly different set-up compared with the ones shown at lectures or in the text book. In other words, they were not able to transfer the procedure they had learned. They did not handle the flexibility potential of the procedures. Rittle-Johnson and Alibali (1999) demonstrated the same phenomenon among children. Their conclusion was that conceptual and procedural knowledge develop iteratively, with gains in one type of knowledge leading to gains in the other.

The introductory statements of not mastering mathematics indicated low self-efficacy. According to Bandura (1997) self-efficacy is about judgements of personal capability within a given activity. Perceived self-efficacy contributes to intellectual performance (Bandura 1997). In his discussion, Bandura (1997) referred to a study showing that children with high self-efficacy in mathematics more quickly discarded faulty strategies, solved more problems, chose to rework more of those they failed, and did so more accurately than children of equal ability, but with lower self-efficacy. Further, children with high self-efficacy were more successful in solving conceptual problems than were children of equal cognitive ability, but lower perceived self-efficacy. It is noteworthy that the respondents in our study only to a modest degree blamed prior teachers or current lecturers for failing their mathematical exams. Instead, they were more prone to blame themselves for not understanding, not daring to ask questions etc. This kind of “self-blaming” confirms low self-efficacy as described by Bandura (1997) and (Tinto 2016).

The majority of respondents reported feeling anxiety both during the mathematical lectures and the exam. They were anxious about being regarded as stupid if they said something during the lectures and they were in particular anxious during the exams. As mentioned by Lyons and Beilock (2011), math anxiety is characterized by feelings of tension and fear and is associated with delayed acquisition of core math and number concepts. The math anxiety is by large a consequence of how students think about their abilities and the nature of the mathematical tasks. Based on their results, Lyons and Beilock (2012) suggest helping students to learn how to control their cognitive processes. Further, math anxiety is associated with low self-efficacy, which has a negative impact on performance (Bandura 1997). People possess the capacity to manage their own thought processes. To the extent that people can control what they think, they can influence how the feel and behave (Bandura 1997, Tice 1995). This mechanism forms the basis of the proposal of Lyons and Beilock (2012) directed towards reducing mathematics anxiety. Control of cognitive processes and high self-efficacy can be achieved by conscious self-talk and goal-setting according to effective methods described by Tice (1995).

The critical incident behind failing the mathematics exam appeared to be absence of understanding of mathematical concepts. The respondents were generally not able to verbalize the meaning or a definition of mathematical concepts. Neither were they able to explain the interrelations between concepts. They admitted that they had never verbalized mathematical concepts. Research on learning of mathematical concepts by verbalizing and discussions, appears to be scant. There are, however, some studies indicating that verbal communication has a substantial impact on understanding and learning. Verbalization helps to place the concept and its content in long-term memory (Solomon, 2007). It has also been demonstrated that verbal training lead to superior concept identification (Klausmeier et al. 1968). Parsons et al. (2005) have developed a procedure on how to communicate about mathematical concepts. I evaluate the following steps as particularly relevant for students: (i) Pronounce the concept, (ii) Define the concept, (iii) Mention a synonym of the concept, and (iv) What is the concept's relationship to other relevant concepts. The potential of "saying" to increase believing and memorizing is supported by the study of Higgins and Rholes (1977). They did not, however, focus on concepts but on evaluation of a person as a consequence of saying something about her/him. Also the hermeneutic spiral may support the importance of "saying" in order to learn. Petzold (1998) delineates a hermeneutic spiral consisting of perception, attention, understanding and explanation. Based on this spiral, it becomes evident that paraphrasing the understanding of a concept may promote learning. This logic is in accordance with the importance of verbalizing mathematical concepts.

Effective intellectual functioning require much more than bits of factual and operational knowledge (Bandura 1997). Additionally it requires meta-cognitive skills for how to organize, monitor, evaluate, and regulate one's thinking processes. According to our respondents, they had not acquired the meta-cognive skills that are needed to manage mathematics. During all their years at school, the mathematical focus had been on procedures and not on the understanding of mathematical concepts. The majority of the respondents expressed that they experienced anxiety concerning mathematics, especially during lectures and exams. This is in accordance with previous studies (Ashcraft and Kirk 2001, Baumeister 1984, Beilock et al. 2004, Lyons and Beilock 2012). Typically, the students were afraid of appearing stupid during the lectures. During the exam, performance anxiety appeared to be linked to loss of self-control.

Mathematics anxiety clearly inhibits math achievements (Ashcraft and Kirk 2001, Lyons and Beilock 2012). The underperformance in mathematics has also been explained by the phenomenon of "stereotype threat" (Schmader and Johns 2003) or "choking under pressure" (Baumeister 1984, Beilock et al. 2004). Additionally to anxiety, several students mentioned that they were embarrassed or ashamed of not managing mathematics. A citation illustrating this embarrassment: "*It is so embarrassing not to understand mathematics in spite of having had it for so many years.*"

Educational implications

The implication is that educational institutions should facilitate learning of mathematics by encouraging the students to verbalize and discuss mathematical concepts in addition to their procedural practicing. By speaking, the concepts are transferred to long-term memory and can be called back to consciousness when needed (Solomon 2007). It is of utmost importance that the practice of verbalizing mathematical concepts is included as an essential part of the students' first year experience. This will form a basis for more successful future learning of mathematics. Generally the usefulness of verbalization of concepts most probably will increase learning in all areas.

Conclusion

It seems that the effect of verbal communication about mathematical concepts in the perspective of learning has not been focused upon in pedagogic research. Our respondents had some standard procedural competence but not conceptual competence. The lack of conceptual understanding seemed to be a critical, underlying cause of failing mathematical exams. Without understanding the concepts, i.e. not being able to make abstractions, they were unable to recognize and solve math tasks that diverged only slightly from the ones they were familiar with through procedural practice. The lack of understanding mathematical concepts was associated with lack of self-control and anxiety at the exam.

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